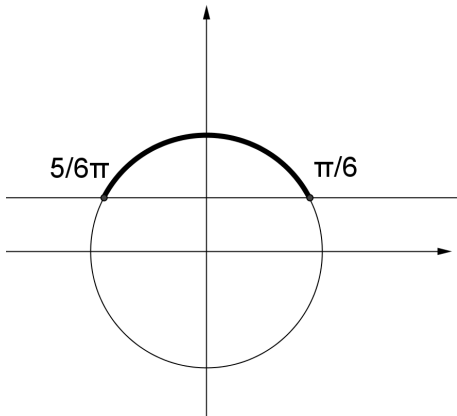


Disequazioni goniometriche

Disequazioni goniometriche elementari

a) Riprendiamo gli esempi che abbiamo fatto per le equazioni trasformandoli in disequazioni:



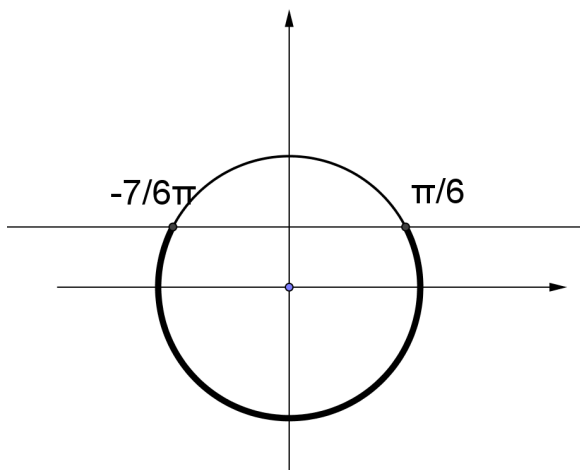
$$\text{sen } x > \frac{1}{2}$$

Le soluzioni saranno:

$$\frac{\pi}{6} + 2k\pi < x < \frac{5}{6}\pi + 2k\pi$$

Se invece devo risolvere:

$$\text{sen } x < \frac{1}{2}$$



le soluzioni possono essere scritte così:

$$-\frac{7}{6}\pi + 2k\pi < x < \frac{\pi}{6} + 2k\pi$$

oppure

$$\frac{5}{6}\pi + 2k\pi < x < \frac{13}{6}\pi + 2k\pi$$

oppure

$$2k\pi \leq x \leq \frac{\pi}{6} + 2k\pi \cup \frac{5}{6}\pi + 2k\pi < x \leq 2\pi + 2k\pi$$

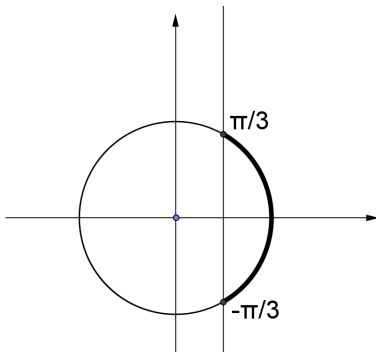
Attenzione: non ha senso scrivere $\frac{5}{6}\pi + 2k\pi < x < \frac{\pi}{6} + 2k\pi$!

Diseguazioni goniometriche

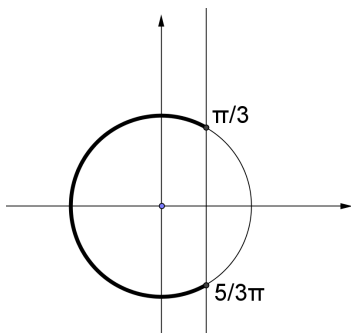
b) $\cos x > \frac{1}{2}$:

le soluzioni sono:

$$-\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi$$

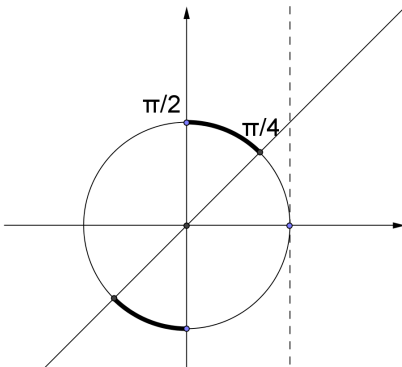


Se invece dovessi risolvere $\cos x < \frac{1}{2}$ avrei:

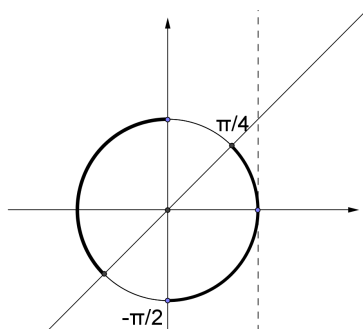


$$\frac{\pi}{3} + 2k\pi < x < \frac{5}{3}\pi + 2k\pi$$

c) $\operatorname{tg} x > 1 \rightarrow \frac{\pi}{4} + k\pi < x < \frac{\pi}{2} + k\pi$



Se invece devo risolvere $\operatorname{tg} x < 1 \rightarrow -\frac{\pi}{2} + k\pi < x < \frac{\pi}{4} + k\pi$



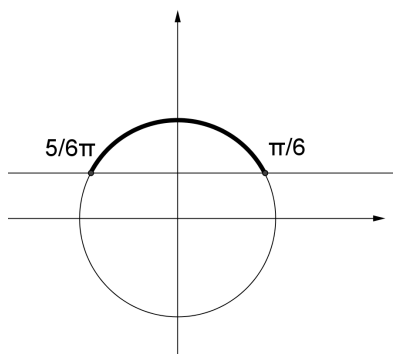
oppure posso scrivere

$$k\pi \leq x < \frac{\pi}{4} + k\pi \cup \frac{\pi}{2} + k\pi < x \leq \pi + k\pi$$

Disequazioni goniometriche riconducibili a disequazioni elementari

1) Come per le equazioni possiamo avere casi così:

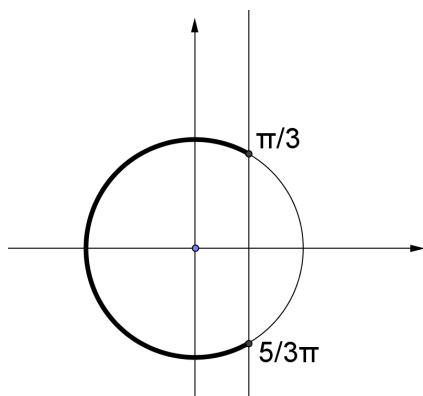
a) $\operatorname{sen}\left(x - \frac{\pi}{4}\right) > \frac{1}{2}$: consideriamo “ $x - \frac{\pi}{4}$ ” tutto insieme:



$$\frac{\pi}{6} + 2k\pi < x - \frac{\pi}{4} < \frac{5}{6}\pi + 2k\pi$$

$$\frac{5}{12}\pi + 2k\pi < x < \frac{13}{12}\pi + 2k\pi$$

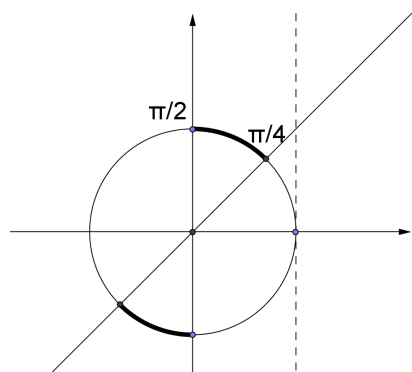
b) $\cos 2x < \frac{1}{2} \rightarrow$



$$\frac{\pi}{3} + 2k\pi < 2x < \frac{5}{3}\pi + 2k\pi$$

$$\frac{\pi}{6} + k\pi < x < \frac{5}{6}\pi + k\pi$$

c) $\operatorname{tg}\left(3x - \frac{\pi}{6}\right) > 1 \rightarrow$



$$\frac{\pi}{4} + k\pi < 3x - \frac{\pi}{6} < \frac{\pi}{2} + k\pi$$

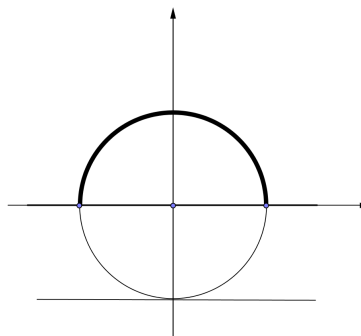
$$\frac{5}{12}\pi + k\pi < 3x < \frac{2}{3}\pi + k\pi$$

$$\frac{5}{36}\pi + k\frac{\pi}{3} < x < \frac{2}{9}\pi + k\frac{\pi}{3}$$

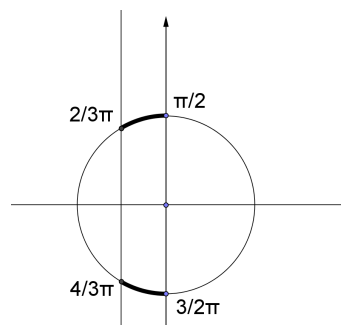
Diseguazioni goniometriche

2)

a) $\text{sen}^2 x + \text{sen} x > 0$
 $\text{sen} x(\text{sen} x + 1) > 0$
 $\text{sen} x < -1 \cup \text{sen} x > 0$
 \Updownarrow
 $2k\pi < x < \pi + 2k\pi$

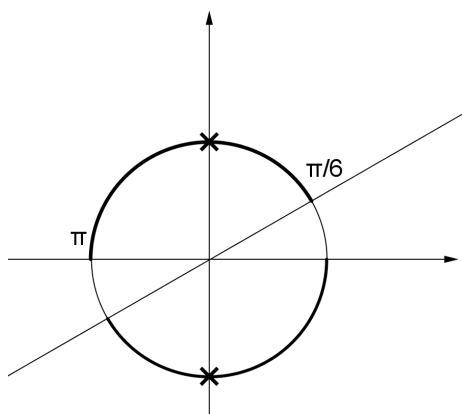


b) $2 \cos^2 x + \cos x < 0$
 $\cos x(2 \cos x + 1) < 0$
 $-\frac{1}{2} < \cos x < 0$



$$\frac{\pi}{2} + 2k\pi < x < \frac{2}{3}\pi + 2k\pi \cup \frac{4}{3}\pi + 2k\pi < x < \frac{3}{2}\pi + 2k\pi$$

c) $\sqrt{3} \text{tg}^2 x + \text{tg} x > 0$
 $\text{tg} x(\sqrt{3} \text{tg} x - 1) > 0$
 $\text{tg} x < 0 \cup \text{tg} x > \frac{1}{\sqrt{3}}$



$$\frac{\pi}{6} + k\pi < x < \pi + k\pi \text{ con } x \neq \frac{\pi}{2} + k\pi$$

d)

$$2(1 - \cos^2 x) - 3 \cos x < 0$$

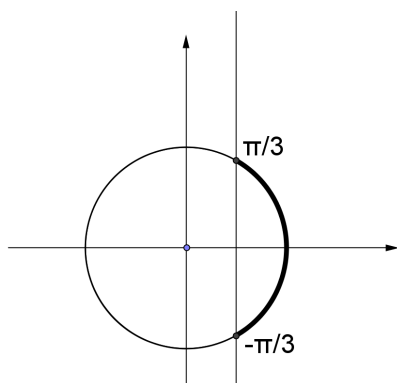
$$2 \cos^2 x + 3 \cos x - 2 > 0$$

$$2 \text{sen}^2 x - 3 \cos x < 0$$

$$\cos x < -2 \cup \cos x > \frac{1}{2}$$

$$(\cos x_{1,2} = \frac{-3 \pm 5}{4} \Rightarrow \cos x = \frac{1}{2} \cup \cos x = -2)$$

$$-\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi$$

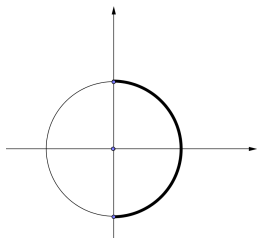


Disequazioni goniometriche

e) $\boxed{\text{sen}2x + \cos x > 0}$
 $2\text{sen}x \cos x + \cos x > 0$
 $\cos x(2\text{sen}x + 1) > 0$

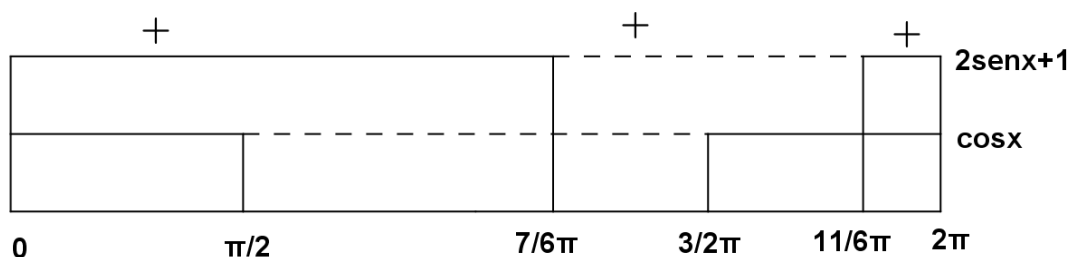
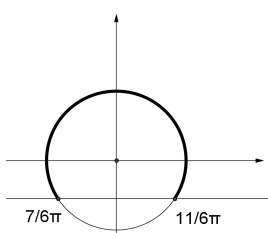
Studio il segno dei singoli fattori del prodotto:

$\cos x > 0 \rightarrow$



$2\text{sen}x + 1 > 0 \rightarrow \text{sen}x > -\frac{1}{2}$

Riporto i risultati tra 0 e 2π



Poiché voglio che il prodotto sia positivo avrò:

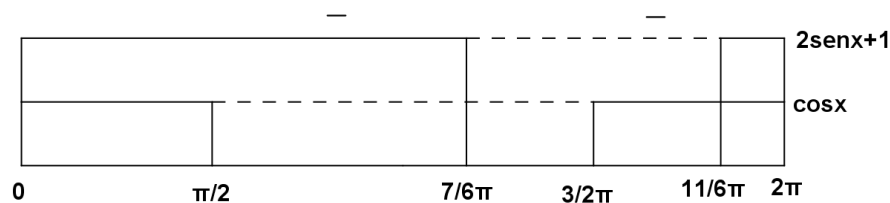
$$2k\pi \leq x < \frac{\pi}{2} + 2k\pi \cup \frac{7}{6}\pi + 2k\pi < x < \frac{3}{2}\pi + 2k\pi \cup \frac{11}{6}\pi + 2k\pi < x \leq 2\pi + 2k\pi$$

f) $\boxed{\frac{\cos x}{2\text{sen}x + 1} < 0}$

Anche in questo caso studio il segno del numeratore e del denominatore e alla fine prendo le zone dove ho una combinazione di segni che mi dà risultato negativo.

N: $\cos x > 0 \rightarrow$ vedi sopra

D: $2\text{sen}x + 1 > 0 \rightarrow$ vedi sopra



$$\frac{\pi}{2} + 2k\pi < x < \frac{7}{6}\pi + 2k\pi \cup \frac{3}{2}\pi + 2k\pi < x < \frac{11}{6}\pi + 2k\pi$$

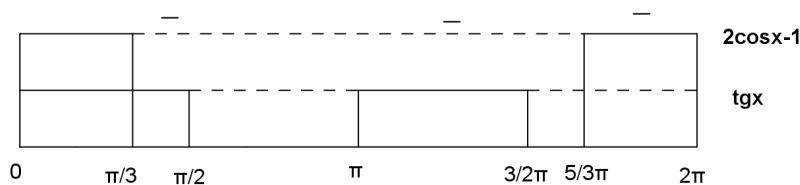
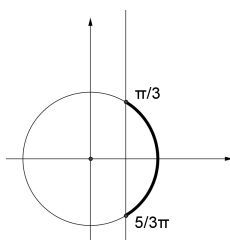
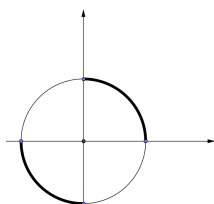
Disequazioni goniometriche

Osservazione: se devo studiare il segno di un prodotto o di un quoziente in cui compaiono funzioni goniometriche di periodo diverso dovrò fare il grafico finale considerando il minimo comune multiplo dei 2 periodi.

Esempio 1: $\frac{\operatorname{tg} x}{2 \cos x - 1} < 0$

N: $\operatorname{tg} x > 0$

D: $2 \cos x - 1 > 0 \rightarrow \cos x > \frac{1}{2}$

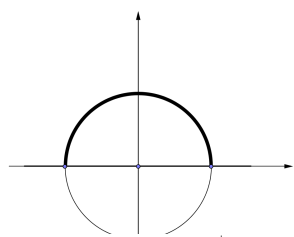


$$\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{2} + 2k\pi \cup \pi + 2k\pi < x < \frac{3}{2}\pi + 2k\pi \cup \frac{5}{3}\pi + 2k\pi < x < 2\pi + 2k\pi$$

Esempio 2: $\operatorname{sen} 3x \cdot \operatorname{tg} x > 0$

Il minimo comune multiplo tra $T_1 = \frac{2}{3}\pi$ e $T_2 = \pi$ è 2π .

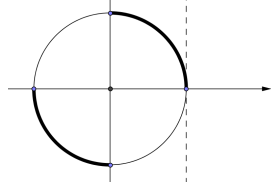
$\operatorname{sen} 3x > 0$



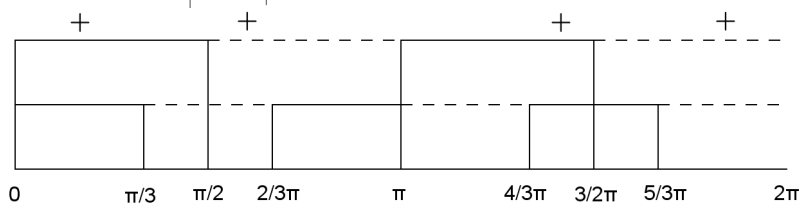
$$2k\pi < 3x < \pi + 2k\pi$$

$$\frac{2}{3}k\pi < x < \frac{\pi}{3} + \frac{2}{3}k\pi$$

$\operatorname{tg} x > 0$



$$k\pi < x < \frac{\pi}{2} + k\pi$$



Disequazioni goniometriche di primo grado in seno e coseno

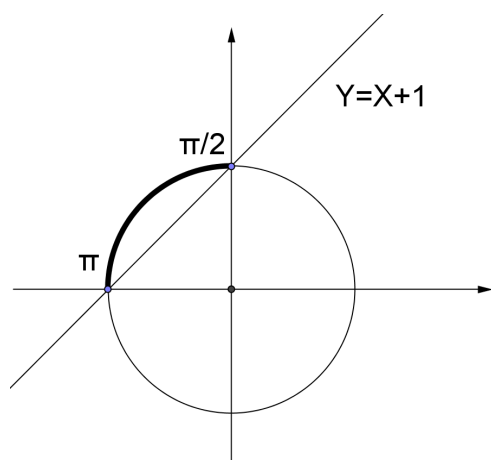
Riprendiamo l'esempio fatto per le equazioni lineari:

$$\boxed{\text{sen}x - \text{cos}x - 1 > 0}$$

(ma questa volta mettiamo >)

Sostituendo $Y = \text{sen}x$ avremo:
 $X = \text{cos}x$

$$\begin{cases} Y - X - 1 > 0 \rightarrow Y > X + 1 \\ X^2 + Y^2 = 1 \end{cases}$$

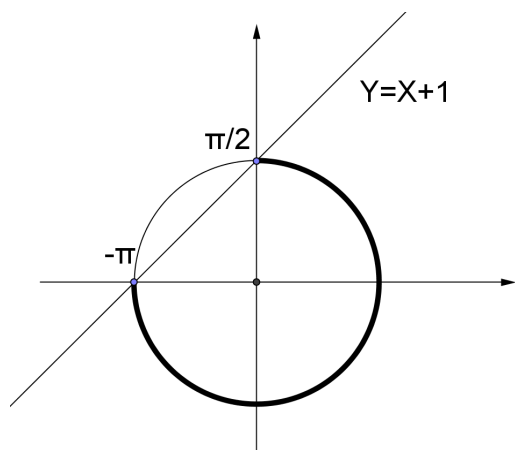


La parte di circonferenza goniometrica in cui i punti hanno $Y > X + 1$ (semipiano "sopra" alla retta $y = x + 1$) è quella indicata in figura e quindi le soluzioni della disequazione sono:

$$\frac{\pi}{2} + 2k\pi < x < \pi + 2k\pi$$

Se avessimo dovuto risolvere:

$$\text{sen}x - \text{cos}x - 1 < 0 \rightarrow \begin{cases} Y < X + 1 \\ X^2 + Y^2 = 1 \end{cases}$$



Le soluzioni possono essere scritte:

$$-\pi + 2k\pi < x < \frac{\pi}{2} + 2k\pi$$

oppure spezzando:

$$2k\pi \leq x < \frac{\pi}{2} + 2k\pi \cup \pi + 2k\pi < x \leq 2\pi + 2k\pi$$

Importante

Mentre per le equazioni goniometriche lineari in cui manca il termine noto avevamo detto che potevamo risolvere anche dividendo per $\cos x$, nel caso delle disequazioni questo non può essere fatto poiché il coseno di un angolo non è sempre positivo.

Consideriamo per esempio la disequazione:

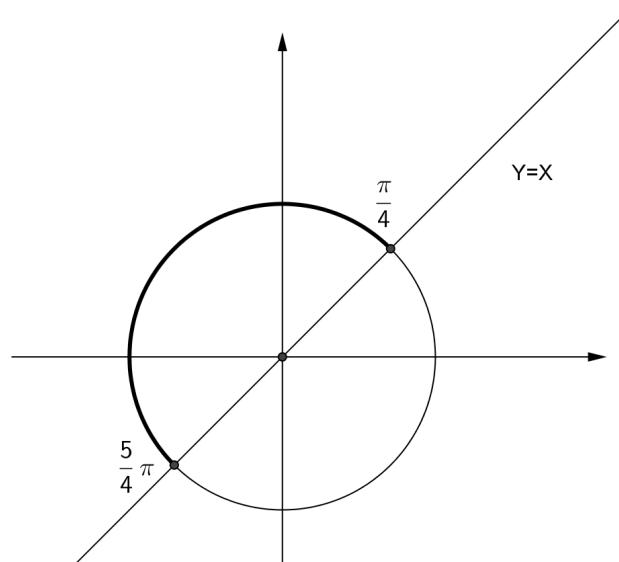
$$\boxed{\text{sen} x - \cos x > 0}$$

Non possiamo dividere per $\cos x$ perché il segno del coseno varia.

Dobbiamo quindi necessariamente applicare il metodo grafico, sostituendo cioè $\text{sen} x = Y$; $\cos x = X$ e intersecando con la circonferenza goniometrica.

$$\begin{cases} Y - X > 0 \\ X^2 + Y^2 = 1 \end{cases} \Rightarrow \begin{cases} Y > X \\ X^2 + Y^2 = 1 \end{cases}$$

Possiamo disegnare facilmente la retta $Y = X$ e quindi considerare in questo caso il semipiano “sopra” alla retta poiché abbiamo $Y > X$: avremo quindi come parte della circonferenza goniometrica quella evidenziata in figura.



In conclusione la soluzione della disequazione sarà data da

$$\frac{\pi}{4} + 2k\pi < x < \frac{5}{4}\pi + 2k\pi$$

Disequazioni goniometriche di secondo grado in seno e coseno omogenee o riconducibili ad omogenee

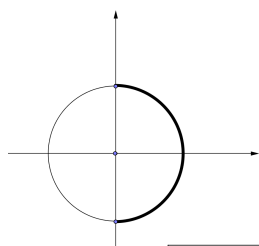
Vediamo infine come si risolvono le disequazioni goniometriche di secondo grado in seno e coseno.

a) $\boxed{\text{sen}x \cdot \cos x + \cos^2 x > 0}$

$$\cos x(\text{sen}x + \cos x) > 0$$

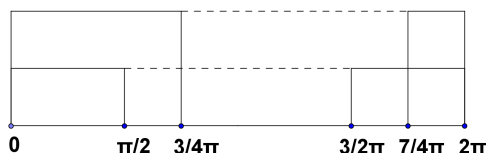
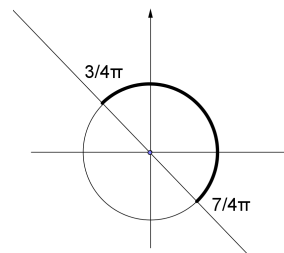
Studio il seno dei singoli fattori:

$$\cos x > 0$$



$$\text{sen}x + \cos x > 0$$

$$\begin{cases} Y + X > 0 \rightarrow Y > -X \\ X^2 + Y^2 = 1 \end{cases}$$



Le soluzioni sono quindi

$$2k\pi \leq x < \frac{\pi}{2} + 2k\pi \cup \frac{3}{4}\pi + 2k\pi < x < \frac{3}{2}\pi + 2k\pi \cup \frac{7}{4}\pi + 2k\pi < x \leq 2\pi + 2k\pi$$

b) $\boxed{\text{sen}^2 x + (1 - \sqrt{3})\text{sen}x \cdot \cos x - \sqrt{3}\cos^2 x < 0}$

1° metodo: sostituiamo a
$$\begin{cases} \text{sen}^2 x = \frac{1 - \cos 2x}{2} \\ \text{sen}x \cdot \cos x = \frac{1}{2}\text{sen}2x \\ \cos^2 x = \frac{1 + \cos 2x}{2} \end{cases}$$

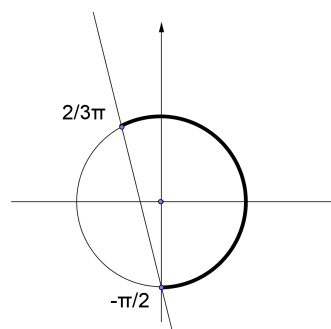
e otteniamo: $\text{sen}2x + (2 + \sqrt{3})\cos 2x + 1 > 0$

Abbiamo quindi ottenuto una disequazione lineare ma l'angolo è $2x$. Poniamo $Y = \text{sen}2x$, $X = \cos 2x$.

Risolviamo.
$$\begin{cases} Y + (2 + \sqrt{3})X + 1 > 0 \dots \\ X^2 + Y^2 = 1 \end{cases}$$

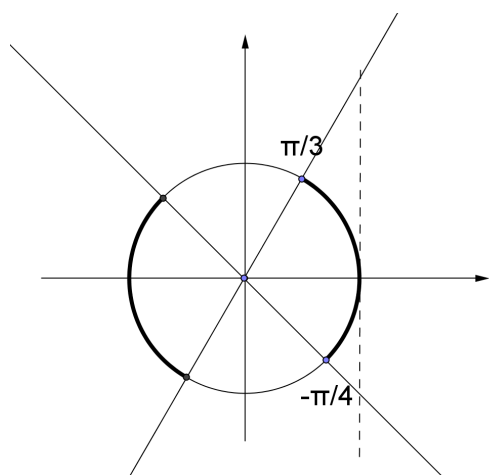
$$-\frac{\pi}{2} + 2k\pi < 2x < \frac{2}{3}\pi + 2k\pi$$

$$-\frac{\pi}{4} + k\pi < x < \frac{\pi}{3} + k\pi$$



Disequazioni goniometriche

2° metodo: dividiamo per $\cos^2 x$ (è positivo) ma controlliamo se $x = \frac{\pi}{2} + k\pi$ è soluzione della disequazione. Sostituendo nell'equazione $x = \frac{\pi}{2} + k\pi$ abbiamo $1+0+0$ che non è minore di 0 e quindi $x = \frac{\pi}{2} + k\pi$ non è soluzione.



$$\operatorname{tg}^2 x + (1 - \sqrt{3}) \operatorname{tg} x - \sqrt{3} < 0 \dots$$

$$-1 < \operatorname{tg} x < \sqrt{3}$$

$$-\frac{\pi}{4} + k\pi < x < \frac{\pi}{3} + k\pi$$

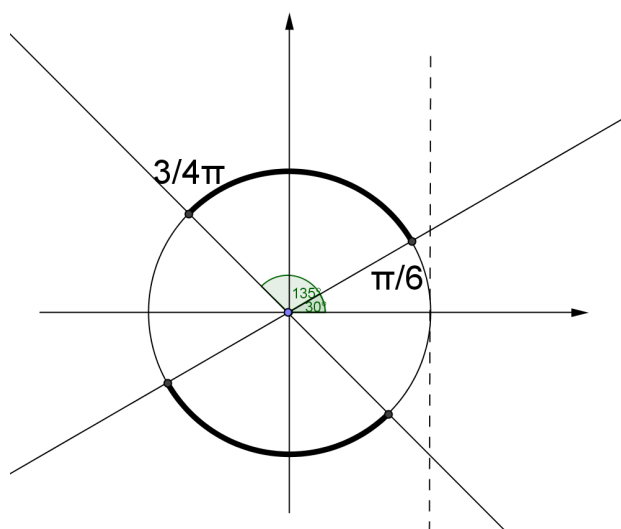
(come infatti avevamo già trovato con il primo metodo)

c)
$$\boxed{(3 + \sqrt{3}) \operatorname{sen}^2 x + 2 \cos^2 x + (\sqrt{3} - 1) \operatorname{sen} x \cdot \cos x > 3}$$

Moltiplichiamo 3 per $1 = \operatorname{sen}^2 x + \cos^2 x$ e dividiamo poi per $\cos^2 x$:
sostituendo $x = \frac{\pi}{2} + k\pi \rightarrow 3 + \sqrt{3}$ ed è maggiore di 3 $\Rightarrow x = \frac{\pi}{2} + k\pi$ è soluzione.

Avremo
$$\sqrt{3} \operatorname{tg}^2 x + (\sqrt{3} - 1) \operatorname{tg} x - 1 > 0$$

$$\operatorname{tg} x < -1 \cup \operatorname{tg} x > \frac{1}{\sqrt{3}}$$



Poiché $x = \frac{\pi}{2} + k\pi$ è soluzione della disequazione possiamo scrivere:

$$\frac{\pi}{6} + k\pi < x < \frac{3}{4}\pi + k\pi$$

ESERCIZI
DISEQUAZIONI GONIOMETRICHE

Riprendiamo gli esercizi sulle equazioni goniometriche e trasformiamoli in disequazioni:

1) Risolvi le seguenti disequazioni goniometriche elementari:

a. $\operatorname{sen} x > -\frac{1}{2}$	$\left[-\frac{\pi}{6} + 2k\pi < x < \frac{7}{6}\pi + 2k\pi \right]$
b. $\cos x > \frac{\sqrt{3}}{2}$	$\left[-\frac{\pi}{6} + 2k\pi < x < \frac{\pi}{6} + 2k\pi \right]$
c. $\operatorname{tg} x > \sqrt{3}$	$\left[\frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi \right]$
d. $\cos x < -\frac{\sqrt{3}}{2}$	$\left[\frac{5}{6}\pi + 2k\pi < x < \frac{7}{6}\pi + 2k\pi \right]$
e. $\operatorname{tg} x < -\frac{1}{\sqrt{3}}$	$\left[-\frac{\pi}{2} + k\pi < x < -\frac{\pi}{6} + k\pi \right]$
f. $\operatorname{sen} x < -\frac{1}{\sqrt{2}}$	$\left[\frac{5}{4}\pi + 2k\pi < x < \frac{7}{4}\pi + 2k\pi \right]$

2) Risolvi le seguenti disequazioni goniometriche riconducibili a disequazioni elementari:

a. $\operatorname{sen}\left(x - \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$	$\left[\frac{\pi}{2} + 2k\pi < x < \pi + 2k\pi \right]$
b. $\cos\left(2x + \frac{\pi}{6}\right) > -\frac{1}{2}$	$\left[-\frac{5}{12} + k\pi < x < \frac{\pi}{4} + k\pi \right]$
c. $\operatorname{tg}\left(x - \frac{\pi}{3}\right) > -1$	$\left[\frac{\pi}{12} + k\pi < x < \frac{5}{6}\pi + k\pi \right]$
d. $\operatorname{sen} 2x < 1$	$\left[x \neq \frac{\pi}{4} + k\pi \right]$
e. $\cos 3x \leq -1$	$\left[x = \frac{\pi}{3} + \frac{2}{3}k\pi \right]$
f. $\operatorname{tg} 4x < -\sqrt{3}$	$\left[-\frac{\pi}{8} + k\frac{\pi}{4} < x < -\frac{\pi}{12} + k\frac{\pi}{4} \right]$

Disequazioni goniometriche

3) Risolvi le seguenti disequazioni goniometriche riconducibili a disequazioni elementari:

a) $\text{sen}^2 x - \text{sen} x > 0$ $[\pi + 2k\pi < x < 2\pi + 2k\pi]$

b) $2\cos^2 x + 3\cos x + 1 > 0$ $[-\frac{2}{3}\pi + 2k\pi < x < \frac{2}{3}\pi + 2k\pi]$

c) $\sqrt{3}\text{tg}^2 x - 4\text{tg} x + \sqrt{3} > 0$ $[-\frac{\pi}{2} + k\pi < x < \frac{\pi}{6} + k\pi \cup \frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi]$

d) $2\cos^2 x - \cos x < 0$ $[-\frac{\pi}{2} + 2k\pi < x < -\frac{\pi}{3} + 2k\pi \cup \frac{\pi}{3} + 2k\pi < x < \frac{\pi}{2} + 2k\pi]$

e) $2\text{sen}^2 x + \text{sen} x < 0$ $[-\frac{\pi}{6} + 2k\pi < x < 2k\pi \cup \pi + 2k\pi < x < \frac{7}{6}\pi + 2k\pi]$

f) $2\text{sen}^2 x - 3\text{sen} x + 1 > 0$ $[-\frac{7}{6} + 2k\pi < x < \frac{\pi}{6} + 2k\pi]$

g) $\text{tg}^2 x + \text{tg} x < 0$ $[-\frac{\pi}{4} + k\pi < x < k\pi]$

h) $\sqrt{3}\text{tg}^2 x + \text{tg} x > 0$ $[k\pi < x < \frac{\pi}{2} + k\pi \cup \frac{\pi}{2} + k\pi < x < \frac{5}{6}\pi + k\pi]$

i) $2\cos^2 x + \cos x - 1 \leq 0$ $[\frac{\pi}{3} + 2k\pi \leq x \leq \frac{5}{3}\pi + 2k\pi]$

l) $3\text{tg}^2 x + 2\sqrt{3}\text{tg} x - 3 \leq 0$ $[-\frac{\pi}{3} + k\pi \leq x \leq \frac{\pi}{6} + k\pi]$

4) Risolvi le seguenti disequazioni goniometriche lineari:

a) $\text{sen} x + \cos x > 1$ $[2k\pi < x < \frac{\pi}{2} + 2k\pi]$

b) $\text{sen} x + \cos x \leq \sqrt{2}$ $[\forall x \in \mathcal{R}]$

c) $\cos x + \sqrt{3}\text{sen} x < \sqrt{3}$ $[-\frac{3}{2}\pi + 2k\pi < x < \frac{\pi}{6} + 2k\pi]$

d) $\sqrt{3}\cos x - \text{sen} x + \sqrt{3} > 0$ $[-\pi + 2k\pi < x < \frac{2}{3}\pi + 2k\pi]$

e) $2\cos x + 2\text{sen} x \geq \sqrt{3} + 1$ $[\frac{\pi}{6} + 2k\pi \leq x \leq \frac{\pi}{3} + 2k\pi]$

f) $\text{sen} x + \cos x > -2$ $[\forall x \in \mathcal{R}]$

Disequazioni goniometriche

5) Risolvi le seguenti disequazioni goniometriche (omogenee o riconducibili ad omogenee) di 2° grado in seno e coseno:

$$\text{a) } 3\text{sen}^2 x - \cos^2 x > 0 \qquad \left[\frac{\pi}{6} + k\pi < x < \frac{5}{6}\pi + k\pi \right]$$

$$\text{b) } \text{sen}^2 x - \text{sen} x \cdot \cos x < 0 \qquad \left[k\pi < x < \frac{\pi}{4} + k\pi \right]$$

$$\text{c) } \sqrt{3} \cos^2 x + \cos x \cdot \text{sen} x > 0 \qquad \left[-\frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi \right]$$

$$\text{d) } 2\text{sen}^2 x - \sqrt{3}\text{sen} x \cdot \cos x - \cos^2 x < \frac{1}{2} \qquad \left[-\frac{\pi}{6} + k\pi < x < \frac{\pi}{3} + k\pi \right]$$

$$\text{e) } 4\text{sen}^2 x - \sqrt{3}\text{sen} x \cdot \cos x + \cos^2 x \leq 1 \qquad \left[k\pi \leq x \leq \frac{\pi}{6} + k\pi \right]$$

$$\text{f) } 2\text{sen}^2 x + \sqrt{3}\text{sen} x \cdot \cos x - \cos^2 x > 2 \qquad \left[\frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi \right]$$

6) Risolvi le seguenti disequazioni goniometriche:

$$\text{a) } 2\cos^2 \frac{x}{2} - \text{sen}^2 \frac{x}{2} > \frac{1}{2} \text{sen}^2 x \qquad \left[-\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{2} + 2k\pi \right]$$

$$\text{b) } (\text{sen} x + \cos x)^2 \geq \cos 2x \qquad \left[k\pi \leq x \leq \frac{3}{4}\pi + k\pi \right]$$

$$\text{c) } \text{tg} 2x - 3\text{tg} x < 0 \qquad \left[k\pi < x < \frac{\pi}{6} + k\pi \cup \frac{\pi}{4} + k\pi < x < \frac{\pi}{2} + k\pi \cup \frac{3}{4}\pi + k\pi < x < \frac{5}{6}\pi + k\pi \right]$$

$$\text{d) } \frac{\text{sen} x - \cos x}{2\cos^2 x - 1} > 0 \qquad \left[\frac{3}{4}\pi + 2k\pi < x < \frac{5}{4}\pi + 2k\pi \cup \frac{5}{4}\pi + 2k\pi < x < \frac{7}{4}\pi + 2k\pi \right]$$

$$\text{e) } \frac{\text{sen} x}{\text{sen} x + \cos x} < 0 \qquad \left[\frac{3}{4}\pi + 2k\pi < x < \pi + 2k\pi \cup \frac{7}{4}\pi + 2k\pi < x < 2\pi + 2k\pi \right]$$

$$\text{f) } \frac{\text{tg}^2 x - 1}{\text{sen} 2x} > 0 \qquad \left[\frac{\pi}{4} + k\pi < x < \frac{\pi}{2} + k\pi \cup \frac{3}{4}\pi + k\pi < x < \pi + k\pi \right]$$

SCHEMA DI VERIFICA
DISEQUAZIONI GONIOMETRICHE

Risolvi le seguenti disequazioni goniometriche:

$$1. \quad 2 \cos\left(x - \frac{\pi}{4}\right) < \sqrt{3} \qquad \left[\frac{5}{12}\pi + 2k\pi < x < \frac{25}{12}\pi + 2k\pi \right]$$

$$2. \quad 4 \operatorname{sen}^2 x < 3 \qquad \left[-\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi \cup \frac{2}{3}\pi + 2k\pi < x < \frac{4}{3}\pi + 2k\pi \right]$$

$$3. \quad \operatorname{tg}^2 x - \sqrt{3} \operatorname{tg} x > 0 \qquad \left[\frac{\pi}{3} + k\pi < x < \pi + k\pi, \quad x \neq \frac{\pi}{2} + k\pi \right]$$

$$4. \quad 2 + 3 \cos x \geq 2 \operatorname{sen}^2 x - 1 \qquad \left[-\frac{2}{3}\pi + 2k\pi \leq x \leq \frac{2}{3}\pi + 2k\pi \cup x = \pi + 2k\pi \right]$$

$$5. \quad 2 \cos^2 x - \cos x < 0 \qquad \left[\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{2} + 2k\pi \cup \frac{3}{2}\pi + 2k\pi < x < \frac{5}{3}\pi + 2k\pi \right]$$

$$6. \quad \frac{\sqrt{3} \operatorname{sen} x - \cos x}{\operatorname{tg}^2 x - 1} < 0 \qquad \left[\frac{\pi}{6} + 2k\pi < x < \frac{\pi}{4} + 2k\pi \cup \frac{3}{4}\pi + 2k\pi < x < \frac{7}{6}\pi + 2k\pi \cup \right. \\ \left. \frac{5}{4}\pi + 2k\pi < x < \frac{7}{4}\pi + 2k\pi, \quad x \neq \frac{3}{2}\pi + 2k\pi \right]$$

$$7. \quad \operatorname{sen}\left(x + \frac{\pi}{3}\right) + \sqrt{3} \cos\left(x + \frac{\pi}{3}\right) > 1 \qquad \left[-\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{6} + 2k\pi \right]$$

$$8. \quad \sqrt{3} \operatorname{sen} 2x > 2 \cos^2 x - 2 \qquad [k\pi < x < \frac{2}{3}\pi + k\pi]$$

SCHEDA PER IL RECUPERO
DISEQUAZIONI GONIOMETRICHE

$$1) \cos x > -\frac{1}{2} \quad \left[-\frac{2}{3}\pi + 2k\pi < x < \frac{2}{3}\pi + 2k\pi \right]$$

$$2) \operatorname{sen} x > -\frac{1}{\sqrt{2}} \quad \left[-\frac{\pi}{4} + 2k\pi < x < \frac{5}{4}\pi + 2k\pi \right]$$

$$3) \operatorname{tg} x > -\frac{1}{\sqrt{3}} \quad \left[-\frac{\pi}{6} + k\pi < x < \frac{\pi}{2} + k\pi \right]$$

$$4) \operatorname{sen} 2x > \frac{\sqrt{3}}{2} \quad \left[\frac{\pi}{6} + k\pi < x < \frac{\pi}{3} + k\pi \right]$$

$$5) \operatorname{tg} 2x < -1 \quad \left[-\frac{\pi}{4} + k\frac{\pi}{2} < x < -\frac{\pi}{8} + k\frac{\pi}{2} \right]$$

$$6) 2\operatorname{sen}^2 x - \operatorname{sen} x < 0 \quad \left[2k\pi < x < \frac{\pi}{6} + 2k\pi \cup \frac{5}{6}\pi + 2k\pi < x < \pi + 2k\pi \right]$$

$$7) 3\operatorname{tg}^2 x - 1 < 0 \quad \left[-\frac{\pi}{6} + k\pi < x < \frac{\pi}{6} + k\pi \right]$$

$$8) 2\cos^2 x - 1 > 0 \quad \left[-\frac{\pi}{4} + 2k\pi < x < \frac{\pi}{4} + 2k\pi \cup \frac{3}{4}\pi + 2k\pi < x < \frac{5}{4}\pi + 2k\pi \right]$$

$$9) \operatorname{tg}^2 x - \operatorname{tg} x < 0 \quad \left[k\pi < x < \frac{\pi}{4} + k\pi \right]$$

$$10) \cos^2 x - \operatorname{sen}^2 x > 0 \quad \left[-\frac{\pi}{4} + k\pi < x < \frac{\pi}{4} + k\pi \right]$$

$$11) \operatorname{sen} x - \cos x < 1 \quad \left[-\pi + 2k\pi < x < \frac{\pi}{2} + 2k\pi \right]$$

$$12) \operatorname{sen} x - \sqrt{3}\cos x < 0 \quad \left[-\frac{2}{3}\pi + 2k\pi < x < \frac{\pi}{3} + 2k\pi \right]$$

$$13) \operatorname{sen} x - \cos x > 0 \quad \left[\frac{\pi}{4} + 2k\pi < x < \frac{5}{4}\pi + 2k\pi \right]$$