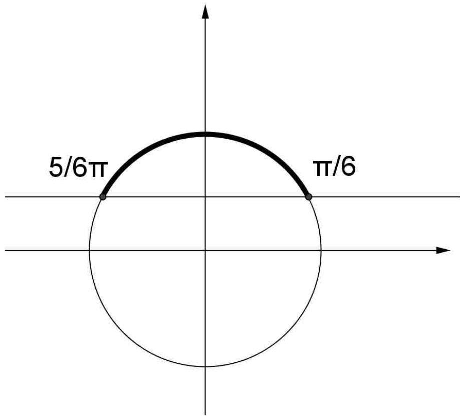
Disequazioni goniometriche

Disequazioni goniometriche elementari

a) Riprendiamo gli esempi che abbiamo fatto per le equazioni trasformandoli in disequazioni:

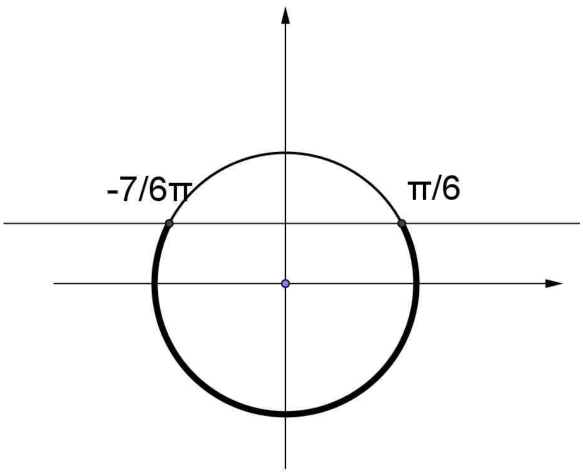


$$senx > \frac{1}{2}$$

Le soluzioni saranno:

$$\frac{\pi}{6} + 2k\pi < x < \frac{5}{6}\pi + 2k\pi$$

Se invece devo risolvere: $senx < \frac{1}{2}$



le soluzioni possono essere scritte così:

$$-\frac{7}{6}\pi + 2k\pi < x < \frac{\pi}{6} + 2k\pi$$

oppure

$$\frac{5}{6}\pi + 2k\pi < x < \frac{13}{6}\pi + 2k\pi$$

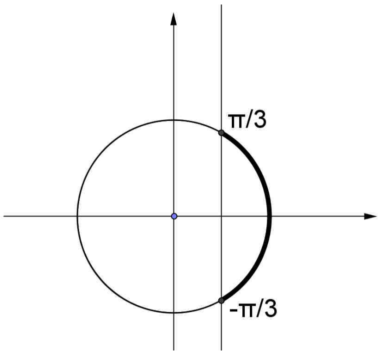
oppure

$$2k\pi \le x \le \frac{\pi}{6} + 2k\pi \cup \frac{5}{6}\pi + 2k\pi < x \le 2\pi + 2k\pi$$

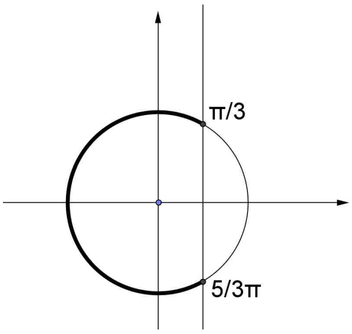
Attenzione: non ha senso scrivere $\frac{5}{6}\pi + 2k\pi < x < \frac{\pi}{6} + 2k\pi$ **!**

b) $\cos x > \frac{1}{2}$: le soluzioni sono:

$$-\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi$$

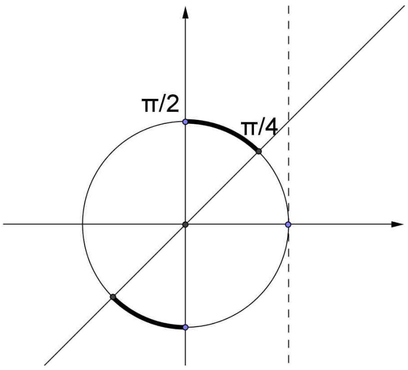


Se invece dovessi risolvere $\cos x < \frac{1}{2}$ avrei:

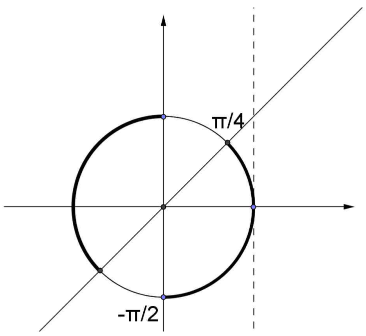


$$\frac{\pi}{3} + 2k\pi < x < \frac{5}{3}\pi + 2k\pi$$

c) $tgx > 1 \rightarrow \frac{\pi}{4} + k\pi < x < \frac{\pi}{2} + k\pi$



Se invece devo risolvere $tgx < 1 \rightarrow -\frac{\pi}{2} + k\pi < x < \frac{\pi}{4} + k\pi$



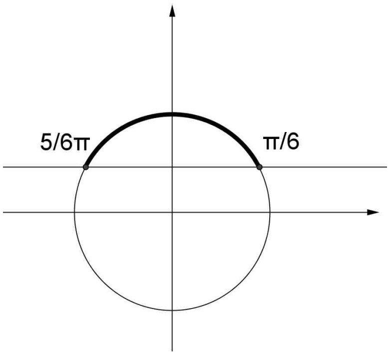
oppure posso scrivere

$$k\pi \le x < \frac{\pi}{4} + k\pi \cup \frac{\pi}{2} + k\pi < x \le \pi + k\pi$$

Disequazioni goniometriche riconducibili a disequazioni elementari

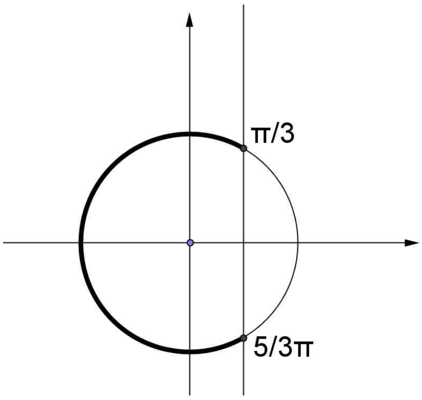
1) Come per le equazioni possiamo avere casi così:

a) $sen\left(x-\frac{\pi}{4}\right)>\frac{1}{2}$: consideriamo “ $x-\frac{\pi}{4}$ ” tutto insieme:



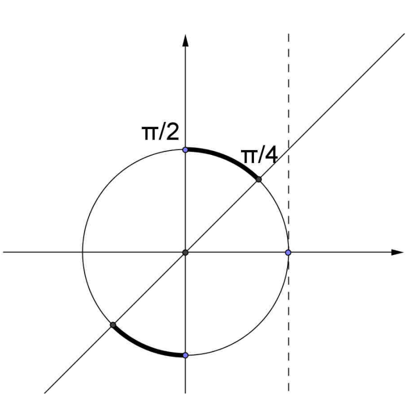
$$\frac{\pi}{6}+2k\pi<x-\frac{\pi}{4}<\frac{5}{6}\pi+2k\pi$$
$$\frac{5}{12}\pi+2k\pi<x<\frac{13}{12}\pi+2k\pi$$

b) $\cos 2x<\frac{1}{2}$ $\rightarrow$



$$\frac{\pi}{3}+2k\pi<2x<\frac{5}{3}\pi+2k\pi$$
$$\frac{\pi}{6}+k\pi<x<\frac{5}{6}\pi+k\pi$$

c) $tg\left(3x-\frac{\pi}{6}\right)>1$ $\rightarrow$



$$\frac{\pi}{4}+k\pi<3x-\frac{\pi}{6}<\frac{\pi}{2}+k\pi$$
$$\frac{5}{12}\pi+k\pi<3x<\frac{2}{3}\pi+k\pi$$
$$\frac{5}{36}\pi+k\frac{\pi}{3}<x<\frac{2}{9}\pi+k\frac{\pi}{3}$$

2)

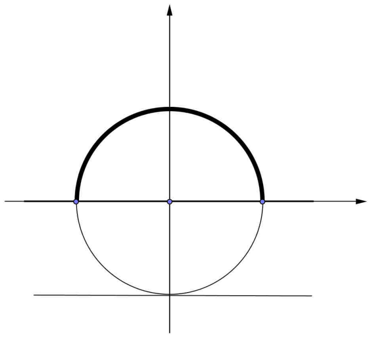
a) $sen^2 x + senx > 0$

$senx(senx+1) > 0$

$senx < -1 \cup senx > 0$

$\Updownarrow$

$2k\pi < x < \pi + 2k\pi$

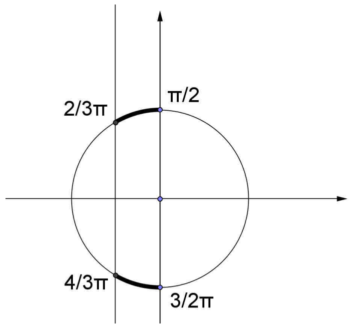


b) $2\cos^2 x + \cos x < 0$

$\cos x(2\cos x + 1) < 0$

$-\frac{1}{2} < \cos x < 0$

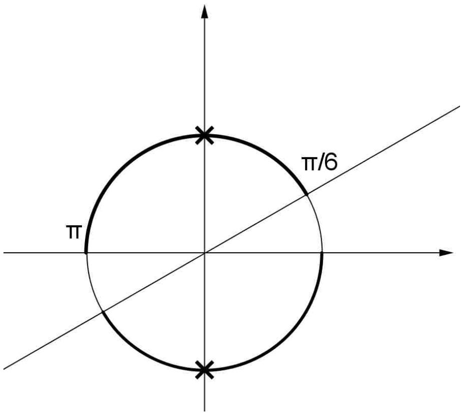
$\frac{\pi}{2} + 2k\pi < x < \frac{2}{3}\pi + 2k\pi \cup \frac{4}{3}\pi + 2k\pi < x < \frac{3}{2}\pi + 2k\pi$



c) $\sqrt{3}tg^2 x + tgx > 0$

$tgx(\sqrt{3}tgx - 1) > 0$

$tgx < 0 \cup tgx > \frac{1}{\sqrt{3}}$



$\frac{\pi}{6} + k\pi < x < \pi + k\pi$ con $x \neq \frac{\pi}{2} + k\pi$

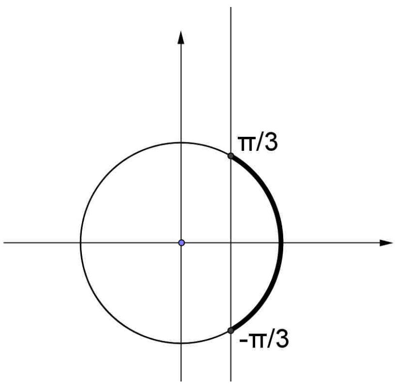
d) $2sen^2 x - 3\cos x < 0$

$2(1-\cos^2 x) - 3\cos x < 0$

$2\cos^2 x + 3\cos x - 2 > 0$

$\cos x < -2 \cup \cos x > \frac{1}{2}$

$(\cos x_{1,2} = \frac{-3 \pm 5}{4} \Rightarrow \cos x = \frac{1}{2} \cup \cos x = -2)$



$-\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi$

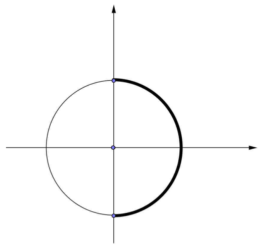
e) $\boxed{sen2x + \cos x > 0}$

$2senx\cos x + \cos x > 0$

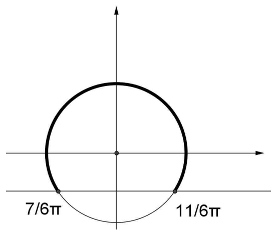
$\cos x(2senx + 1) > 0$

Studio il segno dei singoli fattori del prodotto:

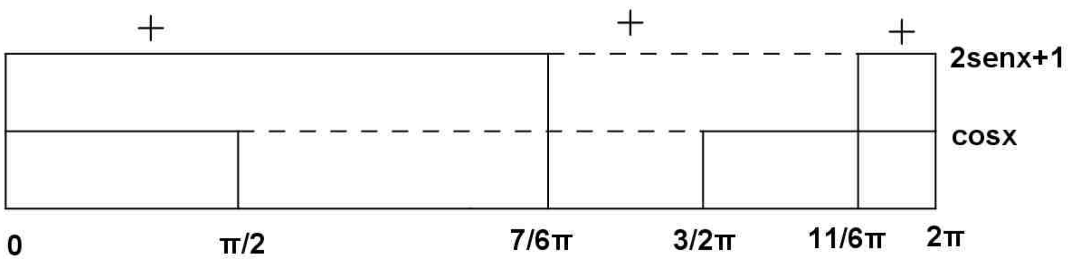
$\cos x > 0 \rightarrow$



$2senx + 1 > 0 \rightarrow senx > -\frac{1}{2}$



Riporto i risultati tra 0 e 2π



Poiché voglio che il prodotto sia positivo avrò:

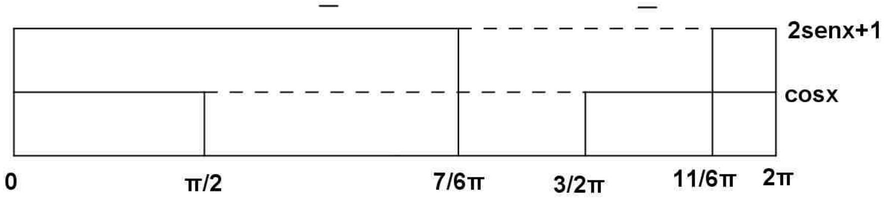
$$2k\pi \le x < \frac{\pi}{2} + 2k\pi \cup \frac{7}{6}\pi + 2k\pi < x < \frac{3}{2}\pi + 2k\pi \cup \frac{11}{6}\pi + 2k\pi < x \le 2\pi + 2k\pi$$

f) $\boxed{\frac{\cos x}{2senx + 1} < 0}$

Anche in questo caso studio il segno del numeratore e del denominatore e alla fine prendo le zone dove ho una combinazione di segni che mi dà risultato negativo.

N: $\cos x > 0 \rightarrow$ vedi sopra

D: $2senx + 1 > 0 \rightarrow$ vedi sopra

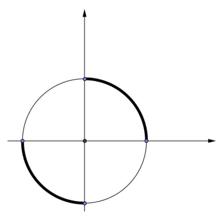


$$\frac{\pi}{2} + 2k\pi < x < \frac{7}{6}\pi + 2k\pi \cup \frac{3}{2}\pi + 2k\pi < x < \frac{11}{6}\pi + 2k\pi$$

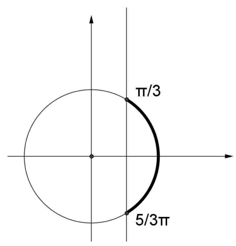
Osservazione: se devo studiare il segno di un prodotto o di un quoziente in cui compaiono funzioni goniometriche di periodo diverso dovrò fare il grafico finale considerando il minimo comune multiplo dei 2 periodi.

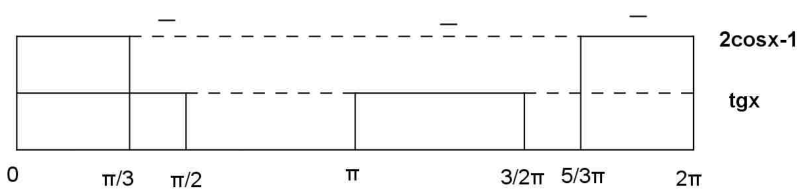
Esempio 1: $\boxed{\dfrac{tgx}{2\cos x-1}<0}$

N: $tgx>0$



D: $2\cos x-1>0 \rightarrow \cos x>\dfrac{1}{2}$



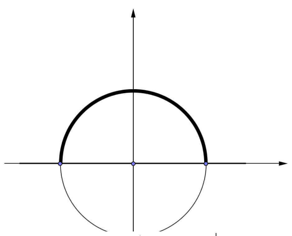


$$\frac{\pi}{3}+2k\pi<x<\frac{\pi}{2}+2k\pi \cup \pi+2k\pi<x<\frac{3}{2}\pi+2k\pi \cup \frac{5}{3}\pi+2k\pi<x<2\pi+2k\pi$$

Esempio 2: $\boxed{sen3x\cdot tgx>0}$

Il minimo comune multiplo tra $T_1=\dfrac{2}{3}\pi$ e $T_2=\pi$ è 2π.

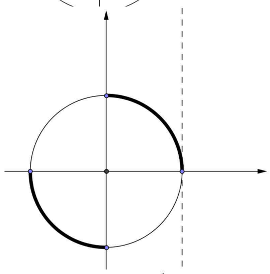
$sen3x>0$



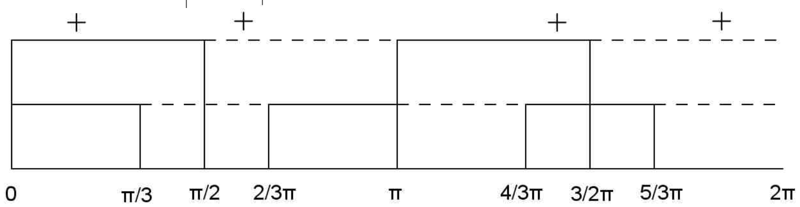
$$2k\pi<3x<\pi+2k\pi$$

$$\frac{2}{3}k\pi<x<\frac{\pi}{3}+\frac{2}{3}k\pi$$

$tgx>0$



$$k\pi<x<\frac{\pi}{2}+k\pi$$



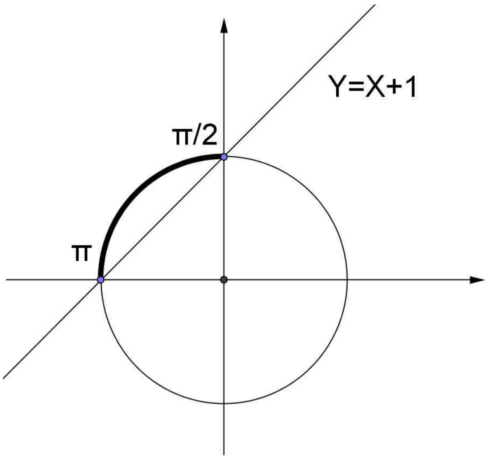
Disequazioni goniometriche di primo grado in seno e coseno

Riprendiamo l'esempio fatto per le equazioni lineari:

$$\boxed{senx - \cos x - 1 > 0}$$ (ma questa volta mettiamo >)

Sostituendo $Y = senx$, $X = \cos x$ avremo:

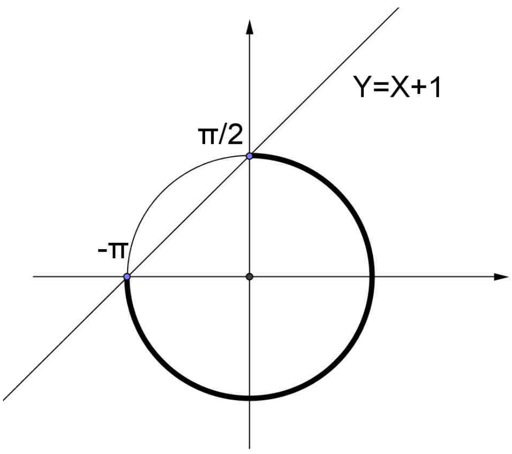
$$\begin{cases} Y - X - 1 > 0 \\ X^2 + Y^2 = 1 \end{cases} \rightarrow \begin{cases} Y > X + 1 \\ X^2 + Y^2 = 1 \end{cases}$$



La parte di circonferenza goniometrica in cui i punti hanno Y>X+1 (semipiano "sopra" alla retta y=x+1) è quella indicata in figura e quindi le soluzioni della disequazione sono:

$$\frac{\pi}{2} + 2k\pi < x < \pi + 2k\pi$$

Se avessimo dovuto risolvere: $$senx - \cos x - 1 < 0 \rightarrow \begin{cases} Y < X + 1 \\ X^2 + Y^2 = 1 \end{cases}$$



Le soluzioni possono essere scritte:

$$-\pi + 2k\pi < x < \frac{\pi}{2} + 2k\pi$$

oppure spezzando:

$$2k\pi \le x < \frac{\pi}{2} + 2k\pi \cup \pi + 2k\pi < x \le 2\pi + 2k\pi$$

Importante

Mentre per le equazioni goniometriche lineari in cui manca il termine noto avevamo detto che potevamo risolvere anche dividendo per cosx, nel caso delle disequazioni questo non può essere fatto poiché il coseno di un angolo non è sempre positivo.

Consideriamo per esempio la disequazione:

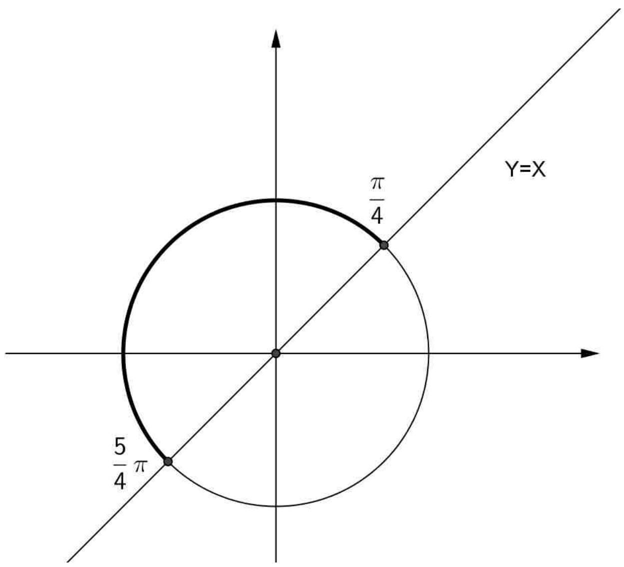
$$\boxed{senx - \cos x > 0}$$

Non possiamo dividere per cosx perché il segno del coseno varia.

Dobbiamo quindi necessariamente applicare il metodo grafico, sostituendo cioè $senx = Y$; $\cos x = X$ e intersecando con la circonferenza goniometrica.

$$\begin{cases} Y - X > 0 \\ X^2 + Y^2 = 1 \end{cases} \Rightarrow \begin{cases} Y > X \\ X^2 + Y^2 = 1 \end{cases}$$

Possiamo disegnare facilmente la retta $Y = X$ e quindi considerare in questo caso il semipiano "sopra" alla retta poiché abbiamo $Y > X$: avremo quindi come parte della circonferenza goniometrica quella evidenziata in figura.



In conclusione la soluzione della disequazione sarà data da

$$\frac{\pi}{4} + 2k\pi < x < \frac{5}{4}\pi + 2k\pi$$

Disequazioni goniometriche di secondo grado in seno e coseno
omogenee o riconducibili ad omogenee

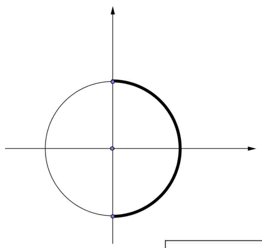
Vediamo infine come si risolvono le disequazioni goniometriche di secondo grado in seno e coseno.

a) $\boxed{senx \cdot \cos x + \cos^2 x > 0}$

$\cos x(senx + \cos x) > 0$

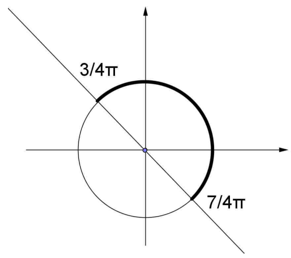
Studio il seno dei singoli fattori:

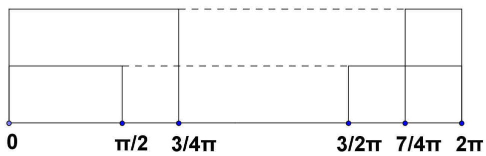
$\cos x > 0$



$senx + \cos x > 0$

$$\begin{cases} Y + X > 0 \rightarrow Y > -X \\ X^2 + Y^2 = 1 \end{cases}$$





Le soluzioni sono quindi

$$2k\pi \le x < \frac{\pi}{2} + 2k\pi \cup \frac{3}{4}\pi + 2k\pi < x < \frac{3}{2}\pi + 2k\pi \cup \frac{7}{4}\pi + 2k\pi < x \le 2\pi + 2k\pi$$

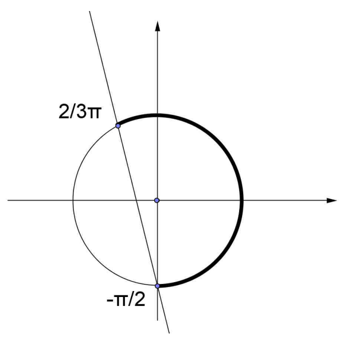
b) $\boxed{sen^2 x + \left(1 - \sqrt{3}\right) senx \cdot \cos x - \sqrt{3}\cos^2 x < 0}$

1° metodo: sostituiamo a $\begin{cases} sen^2 x = \dfrac{1 - \cos 2x}{2} \\ senx \cdot \cos x = \dfrac{1}{2} sen2x \\ \cos^2 x = \dfrac{1 + \cos 2x}{2} \end{cases}$

e otteniamo: $sen2x + \left(2 + \sqrt{3}\right)\cos 2x + 1 > 0$

Abbiamo quindi ottenuto una disequazione lineare ma l'angolo è 2x. Poniamo $Y = sen2x$, $X = \cos 2x$.

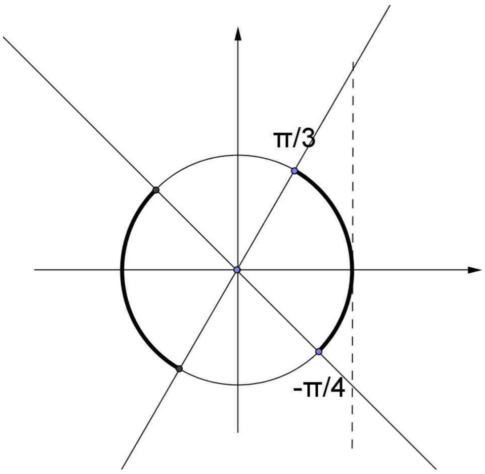
Risolviamo. $\begin{cases} Y + \left(2 + \sqrt{3}\right)X + 1 > 0 ..... \\ X^2 + Y^2 = 1 \end{cases}$



$$-\frac{\pi}{2} + 2k\pi < 2x < \frac{2}{3}\pi + 2k\pi$$

$$-\frac{\pi}{4} + k\pi < x < \frac{\pi}{3} + k\pi$$

2° metodo: dividiamo per $\cos^2 x$ (è positivo) ma controlliamo se $x = \frac{\pi}{2} + k\pi$ è soluzione della disequazione. Sostituendo nell'equazione $x = \frac{\pi}{2} + k\pi$ abbiamo $1 + 0 + 0$ che non è minore di 0 e quindi $x = \frac{\pi}{2} + k\pi$ non è soluzione.



$$tg^2 x + (1 - \sqrt{3}) tgx - \sqrt{3} < 0 \ \ldots$$

$$-1 < tgx < \sqrt{3}$$

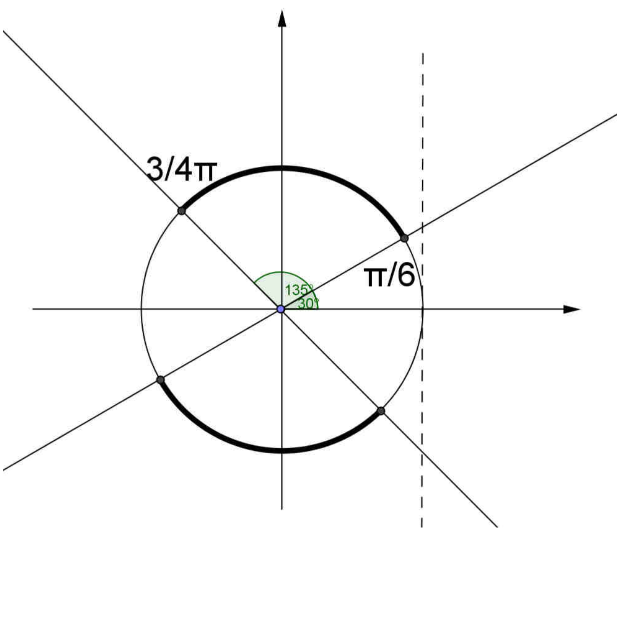
$$-\frac{\pi}{4} + k\pi < x < \frac{\pi}{3} + k\pi$$

(come infatti avevamo già trovato con il primo metodo)

c) $\boxed{(3 + \sqrt{3}) sen^2 x + 2\cos^2 x + (\sqrt{3} - 1) senx \cdot \cos x > 3}$

Moltiplichiamo 3 per 1 $= sen^2 x + \cos^2 x$ e dividiamo poi per $\cos^2 x$: sostituendo $x = \frac{\pi}{2} + k\pi \rightarrow 3 + \sqrt{3}$ ed è maggiore di $3 \Rightarrow x = \frac{\pi}{2} + k\pi$ è soluzione.

Avremo
$$\sqrt{3} tg^2 x + (\sqrt{3} - 1) tgx - 1 > 0$$
$$tgx < -1 \cup tgx > \frac{1}{\sqrt{3}}$$



Poiché $x = \frac{\pi}{2} + k\pi$ è soluzione della disequazione possiamo scrivere:

$$\frac{\pi}{6} + k\pi < x < \frac{3}{4}\pi + k\pi$$

ESERCIZI
DISEQUAZIONI GONIOMETRICHE

Riprendiamo gli esercizi sulle equazioni goniometriche e trasformiamoli in disequazioni:

1) Risolvi le seguenti disequazioni goniometriche elementari:

a. $senx > -\frac{1}{2}$ $\left[-\frac{\pi}{6}+2k\pi < x < \frac{7}{6}\pi + 2k\pi\right]$

b. $\cos x > \frac{\sqrt{3}}{2}$ $\left[-\frac{\pi}{6}+2k\pi < x < \frac{\pi}{6} + 2k\pi\right]$

c. $tgx > \sqrt{3}$ $\left[\frac{\pi}{3}+k\pi < x < \frac{\pi}{2} + k\pi\right]$

d. $\cos x < -\frac{\sqrt{3}}{2}$ $\left[\frac{5}{6}\pi+2k\pi < x < \frac{7}{6} + 2k\pi\right]$

e. $tgx < -\frac{1}{\sqrt{3}}$ $\left[-\frac{\pi}{2}+k\pi < x < -\frac{\pi}{6} + k\pi\right]$

f. $senx < -\frac{1}{\sqrt{2}}$ $\left[\frac{5}{4}\pi+2k\pi < x < \frac{7}{4} + 2k\pi\right]$

2) Risolvi le seguenti disequazioni goniometriche riconducibili a disequazioni elementari:

a. $sen\left(x-\frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$ $\left[\frac{\pi}{2}+2k\pi < x < \pi + 2k\pi\right]$

b. $\cos\left(2x+\frac{\pi}{6}\right) > -\frac{1}{2}$ $\left[-\frac{5}{12}+k\pi < x < \frac{\pi}{4} + k\pi\right]$

c. $tg\left(x-\frac{\pi}{3}\right) > -1$ $\left[\frac{\pi}{12}+k\pi < x < \frac{5}{6}\pi + k\pi\right]$

d. $sen2x < 1$ $\left[x \neq \frac{\pi}{4} + k\pi\right]$

e. $\cos\cdot 3x \leq -1$ $\left[x = \frac{\pi}{3} + \frac{2}{3}k\pi\right]$

f. $tg4x < -\sqrt{3}$ $\left[-\frac{\pi}{8}+k\frac{\pi}{4} < x < -\frac{\pi}{12} + k\frac{\pi}{4}\right]$

3) Risolvi le seguenti disequazioni goniometriche riconducibili a disequazioni elementari:

a) $sen^2x - senx > 0$ $[\pi + 2k\pi < x < 2\pi + 2k\pi\ ]$

b) $2\cos^2 x + 3\cos x + 1 > 0$ $[-\frac{2}{3}\pi + 2k\pi < x < \frac{2}{3}\pi + 2k\pi]$

c) $\sqrt{3}tg^2x - 4tgx + \sqrt{3} > 0$ $[-\frac{\pi}{2} + k\pi < x < \frac{\pi}{6} + k\pi \cup \frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi]$

d) $2\cos^2 x - \cos x < 0$ $[-\frac{\pi}{2} + 2k\pi < x < -\frac{\pi}{3} + 2k\pi \cup \frac{\pi}{3} + 2k\pi < x < \frac{\pi}{2} + 2k\pi]$

e) $2sen^2x + senx < 0$ $[-\frac{\pi}{6} + 2k\pi < x < 2k\pi \cup \pi + 2k\pi < x < \frac{7}{6}\pi + 2k\pi]$

f) $2sen^2x - 3senx + 1 > 0$ $[-\frac{7}{6} + 2k\pi < x < \frac{\pi}{6} + 2k\pi]$

g) $tg^2x + tgx < 0$ $[-\frac{\pi}{4} + k\pi < x < k\pi]$

h) $\sqrt{3}tg^2x + tgx > 0$ $[k\pi < x < \frac{\pi}{2} + k\pi \cup \frac{\pi}{2} + k\pi < x < \frac{5}{6}\pi + k\pi]$

i) $2\cos^2 x + \cos x - 1 \leq 0$ $[\frac{\pi}{3} + 2k\pi \leq x \leq \frac{5}{3}\pi + 2k\pi]$

l) $3tg^2x + 2\sqrt{3}tgx - 3 \leq 0$ $[-\frac{\pi}{3} + k\pi \leq x \leq \frac{\pi}{6} + k\pi]$

4) Risolvi le seguenti disequazioni goniometriche lineari:

a) $senx + \cos x > 1$ $[2k\pi < x < \frac{\pi}{2} + 2k\pi]$

b) $senx + \cos x \leq \sqrt{2}$ $[\forall x \in R]$

c) $\cos x + \sqrt{3}senx < \sqrt{3}$ $[-\frac{3}{2}\pi + 2k\pi < x < \frac{\pi}{6} + 2k\pi]$

d) $\sqrt{3}\cos x - senx + \sqrt{3} > 0$ $[-\pi + 2k\pi < x < \frac{2}{3}\pi + 2k\pi]$

e) $2\cos x + 2senx \geq \sqrt{3} + 1$ $[\frac{\pi}{6} + 2k\pi \leq x \leq \frac{\pi}{3} + 2k\pi]$

f) $senx + \cos x > -2$ $[\forall x \in R\ ]$

5) Risolvi le seguenti disequazioni goniometriche (omogenee o riconducibili ad omogenee) di 2° grado in seno e coseno:

a) $3sen^2 x - \cos^2 x > 0$ $\quad [\frac{\pi}{6} + k\pi < x < \frac{5}{6}\pi + k\pi]$

b) $sen^2 x - senx \cdot \cos x < 0$ $\quad [k\pi < x < \frac{\pi}{4} + k\pi]$

c) $\sqrt{3}\cos^2 x + \cos x \cdot senx > 0$ $\quad [-\frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi]$

d) $2sen^2 x - \sqrt{3}senx \cdot \cos x - \cos^2 x < \frac{1}{2}$ $\quad [-\frac{\pi}{6} + k\pi < x < \frac{\pi}{3} + k\pi]$

e) $4sen^2 x - \sqrt{3}senx \cdot \cos x + \cos^2 x \leq 1$ $\quad [k\pi \leq x \leq \frac{\pi}{6} + k\pi]$

f) $2sen^2 x + \sqrt{3}senx \cdot \cos x - \cos^2 x > 2$ $\quad [\frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi]$

6) Risolvi le seguenti disequazioni goniometriche:

a) $2\cos^2 \frac{x}{2} - sen^2 \frac{x}{2} > \frac{1}{2} sen^2 x$ $\quad [-\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{2} + 2k\pi]$

b) $(senx + \cos x)^2 \geq \cos 2x$ $\quad [k\pi \leq x \leq \frac{3}{4}\pi + k\pi]$

c) $tg2x - 3tgx < 0$ $\quad [k\pi < x < \frac{\pi}{6} + k\pi \cup \frac{\pi}{4} + k\pi < x < \frac{\pi}{2} + k\pi \cup \frac{3}{4}\pi + k\pi < x < \frac{5}{6}\pi + k\pi]$

d) $\dfrac{senx - \cos x}{2\cos^2 x - 1} > 0$

$[\frac{3}{4}\pi + 2k\pi < x < \frac{5}{4}\pi + 2k\pi \cup \frac{5}{4}\pi + 2k\pi < x < \frac{7}{4}\pi + 2k\pi]$

e) $\dfrac{senx}{senx + \cos x} < 0$ $\quad [\frac{3}{4}\pi + 2k\pi < x < \pi + 2k\pi \cup \frac{7}{4}\pi + 2k\pi < x < 2\pi + 2k\pi]$

f) $\dfrac{tg^2 x - 1}{sen2x} > 0$ $\quad [\frac{\pi}{4} + k\pi < x < \frac{\pi}{2} + k\pi \cup \frac{3}{4}\pi + k\pi < x < \pi + k\pi]$

SCHEDA DI VERIFICA
DISEQUAZIONI GONIOMETRICHE

Risolvi le seguenti disequazioni goniometriche:

1. $2\cos(x-\frac{\pi}{4})<\sqrt{3}$ $\quad [\frac{5}{12}\pi+2k\pi<x<\frac{25}{12}\pi+2k\pi]$

2. $4sen^2x<3$ $\quad [-\frac{\pi}{3}+2k\pi<x<\frac{\pi}{3}+2k\pi \cup \frac{2}{3}\pi+2k\pi<x<\frac{4}{3}\pi+2k\pi]$

3. $tg^2x-\sqrt{3}tgx>0$ $\quad [\frac{\pi}{3}+k\pi<x<\pi+k\pi \;,\; x\neq\frac{\pi}{2}+k\pi]$

4. $2+3\cos x\geq 2sen^2x-1$ $\quad [-\frac{2}{3}\pi+2k\pi\leq x\leq\frac{2}{3}\pi+2k\pi \cup x=\pi+2k\pi]$

5. $2\cos^2x-\cos x<0$ $\quad [\frac{\pi}{3}+2k\pi<x<\frac{\pi}{2}+2k\pi \cup \frac{3}{2}\pi+2k\pi<x<\frac{5}{3}\pi+2k\pi]$

6. $\frac{\sqrt{3}senx-\cos x}{tg^2x-1}<0$ $\quad [\frac{\pi}{6}+2k\pi<x<\frac{\pi}{4}+2k\pi \cup \frac{3}{4}\pi+2k\pi<x<\frac{7}{6}\pi+2k\pi \cup$
$\frac{5}{4}\pi+2k\pi<x<\frac{7}{4}\pi+2k\pi, x\neq\frac{3}{2}\pi+2k\pi]$

7. $sen(x+\frac{\pi}{3})+\sqrt{3}\cos(x+\frac{\pi}{3})>1$ $\quad [-\frac{\pi}{2}+2k\pi<x<\frac{\pi}{6}+2k\pi]$

8. $\sqrt{3}sen2x>2\cos^2x-2$ $\quad [k\pi<x<\frac{2}{3}\pi+k\pi]$

SCHEDA PER IL RECUPERO
DISEQUAZIONI GONIOMETRICHE

1) $\cos x > -\frac{1}{2}$ $\left[-\frac{2}{3}\pi + 2k\pi < x < \frac{2}{3}\pi + 2k\pi\right]$

2) $senx > -\frac{1}{\sqrt{2}}$ $\left[-\frac{\pi}{4} + 2k\pi < x < \frac{5}{4}\pi + 2k\pi\right]$

3) $tgx > -\frac{1}{\sqrt{3}}$ $\left[-\frac{\pi}{6} + k\pi < x < \frac{\pi}{2} + k\pi\right]$

4) $sen2x > \frac{\sqrt{3}}{2}$ $\left[\frac{\pi}{6} + k\pi < x < \frac{\pi}{3} + k\pi\right]$

5) $tg2x < -1$ $\left[-\frac{\pi}{4} + k\frac{\pi}{2} < x < -\frac{\pi}{8} + k\frac{\pi}{2}\right]$

6) $2sen^2 x - senx < 0$ $\left[2k\pi < x < \frac{\pi}{6} + 2k\pi \quad \cup \quad \frac{5}{6}\pi + 2k\pi < x < \pi + 2k\pi\right]$

7) $3tg^2 x - 1 < 0$ $\left[-\frac{\pi}{6} + k\pi < x < \frac{\pi}{6} + k\pi\right]$

8) $2\cos^2 x - 1 > 0$ $\left[-\frac{\pi}{4} + 2k\pi < x < \frac{\pi}{4} + 2k\pi \quad \cup \quad \frac{3}{4}\pi + 2k\pi < x < \frac{5}{4}\pi + 2k\pi\right]$

9) $tg^2 x - tgx < 0$ $\left[k\pi < x < \frac{\pi}{4} + k\pi\right]$

10) $\cos^2 x - sen^2 x > 0$ $\left[-\frac{\pi}{4} + k\pi < x < \frac{\pi}{4} + k\pi\right]$

11) $senx - \cos x < 1$ $\left[-\pi + 2k\pi < x < \frac{\pi}{2} + 2k\pi\right]$

12) $senx - \sqrt{3}\cos x < 0$ $\left[-\frac{2}{3}\pi + 2k\pi < x < \frac{\pi}{3} + 2k\pi\right]$

13) $senx - \cos x > 0$ $\left[\frac{\pi}{4} + 2k\pi < x < \frac{5}{4}\pi + 2k\pi\right]$